

Asymptotic Freedom and QCD Scaling Law

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We present a derivation of the QCD scaling law on the basis of asymptotic freedom

The pioneering investigations [1, 2] in QCD scaling laws were marked by an instructive combination of the techniques of “light cone and renormalization group” (hereafter called LC-RG). The resulting structure function formulas worked quite well for large x but small x posed one serious analytical problem during “inversion”. Yet, it was these very works that made the asymptotically free gauge theory suitable for experimental test. Another remarkable development followed when the so-called “anomalous dimension” occurring in the moment equation of the preceding papers was related to the Mellin Transform of the quark distribution. The resulting equation [3] came to be known as the QCD master equation.

So far so good. But, then the historical importance of these pattern-setting works has almost blindfolded us about any alternative possibility of deriving the QCD scaling behaviour without using the results of LC-RG. The present paper derives an analytical expression for nonsinglet structure function which brings out the notable features of QCD prediction for both large and small x . The only physical assumption required for our work is that the strong coupling parameter becomes insensitive to Q^2 variation as it approaches the region of asymptotic freedom. The rest is just rigorous application of Debye’s steepest descent method. It is rigour that simplifies the final formula besides overcoming the trouble “sub-dominant contributions” in case x is small. We begin with the QCD master equation and employ Debye’s method. It is necessary to stress that unlike in [3], the master equation can be derived without using LC-RG⁴ and hence our use of it does not in-

volve us in LC-RG. In this note we intend to demonstrate the fact that the scaling behaviour is built in inside the notion of asymptotic freedom, and other physical ingredients are not required to disclose that behaviour.

We reiterate that our concern in this brief note is the nonsinglet combination of structure functions. (Work is in progress for the general case which will include the singlet case also.) For nonsinglet structure function the coupled evolution equation (master equation) is not required and hence our evolution equation is simple and is given by

$$\frac{dq(x, t)}{dt} = \frac{\alpha(t)}{2} \int_x^1 \frac{dy}{y} q(y, Q^2) P(x/y), \quad (1)$$

where $t = \ln Q^2/Q_0^2$ and $P(x/y)$ implies $P_{q \rightarrow q}(x/y)$. Taking Mellin transform of both sides of (1) we have

$$\frac{dM_q(s, t)}{dt} = a(t) M_p(s) M_q(s, t), \quad (2)$$

where M_q, M_p are respectively the Mellin transforms of q and p defined by

$$M_q(s, t) = \int_0^1 dx x^s q(x, t), \quad (3)$$

$$M_p(s) = \int_0^1 dz z^s p(z), \quad (4)$$

and

$$a(t) = \frac{\alpha(t)}{2\pi}. \quad (5)$$

In deriving (2) we have used the convolution theorem of Mellin transform. Equation (2) can be solved easily and we obtain

$$M_q(s, t) = c \exp(M_p(s) a t). \quad (6)$$

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In deriving (6) we have used the condition of approach to the asymptotic region by taking

$$\int a(t) dt \simeq at. \quad (7)$$

If an input distribution $q(x, 0) \sim (1-x)^d$ be used in the definition (3) we get

$$M_q(s, 0) = \int_0^1 dx x^s q(x, 0) = B(s+1, d+1). \quad (8)$$

Comparing (8) and (6) we can replace c by $M_q(s, 0)$ and write

$$M_q(s, t) = B(s+1, d+1) \exp(M_p(s) at). \quad (9)$$

If we take $P(z) = \frac{4}{3} \frac{1+z^2}{1-z}$, then $M_p(s)$ is given by

$$M_p(s) = C_2(R) \left[\frac{3}{2} + \frac{1}{(s+1)(s+2)} - 2\psi(s+2) - 2\gamma \right], \quad (10)$$

where $c_2(R)$ is a Casimir operator for the adjoint representation G of the colour group, $\psi(s)$ the logarithmic gamma function and γ the Euler constant defined by

$$\gamma = \lim_{n \rightarrow \infty} \left[\frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{n} - \log n \right]. \quad (11)$$

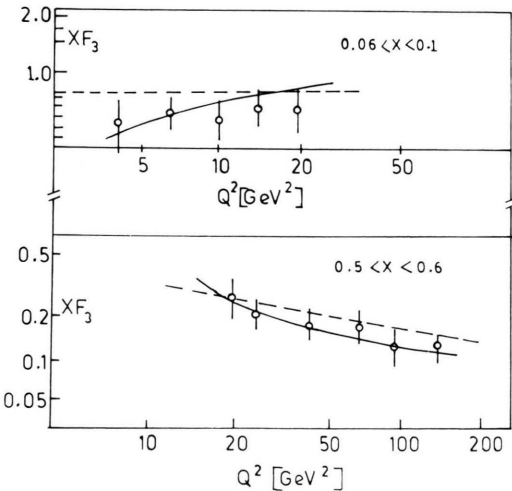


Fig. 1. Comparison of $x F_3(x, Q^2)$ with CDHS data given in Fig. 13 ([4]) continuous curves represents present work. Above small x ($= 0.1$), below large x ($= 0.5$).

Our inversion of (9) by Debye's method begins with

$$q(x, t) = \frac{1}{2\pi i} \left[\int_c \exp[h f(s, x)] B(s+1, d+1) ds \right], \quad (12)$$

where

$$f(s, x) = M_p(s) - (s+1) \frac{\ln x}{at} \quad (13)$$

and $h = at$.

The contour c of (12) is such that on a part c_0 of it the Debye conditions hold. Without dwelling on analyticity details we just point out that unlike in the nonrigorous versions we use a full expansion of $f(s)$ about s_0 , the saddle point

$$f(s) = f(s_0) - u^2, \quad (14)$$

whereby the Debye conditions u is real and where $\text{Ref}(s)$ registers a steep fall with the rise of u . From above it is easy to see that (14) allows a good approximation of (12) in the following form.

$$q(x, t) = \exp(h f(s_0)) \frac{1}{2\pi i} \int_{-\infty}^{\infty} \exp(-h u^2) B(s+1, d+1) \frac{ds}{du}(u) du. \quad (15)$$

Using the power series expansion

$$\frac{1}{2\pi i} B(s+1, d+1) \frac{ds}{du}(u) = \sum_{n=0}^{\infty} a_n u^n$$

we obtain from (15)

$$\begin{aligned} q(x, t) &= \exp(h f(s_0)) \sum_{n=0}^{\infty} a_n \int_{-\infty}^{\infty} \exp(-h u^2) u^n du \\ &= \exp(h f(s_0)) \sum_{n=0}^{\infty} a_{2n} \Gamma(n+1/2) \cdot h^{-(n+1/2)}. \end{aligned} \quad (16)$$

But scaling law experiments imply large t ($t = \ln Q^2/Q_0^2$), i.e. large h , and the factor $h^{-(n+1/2)}$ of (17) indicates that the lowest order approximation is good enough for the present purpose. To the lowest order, we have from (17)

$$q(x, t) = a_0 (\pi/t)^{1/2} \exp(at M_p(s_0)) (1/x)^{s_0+1}, \quad (18)$$

where s_0 given by the equation

$$\left(\frac{\delta f}{\delta s} \right)_{s=s_0} = 0. \quad (19)$$

Since the higher values of n in (17) are forbidden in the method adopted in the present work, the simple looking formula (18) is not open to question re-

garding uniform convergence in n of higher order terms in Wilson Coefficients and anomalous dimensions. This explains why (18) is valid even for small x . If q be taken to represent the distribution of quark minus antiquark, (18) itself represents $x F_3$.

To compare our results with experiment we calculate $q(x, t)$ according to the formula given in (18). In Fig. 1 our theoretical calculation is compared with the experimental result for small x ($x = .1$).

According to (18) a slow rise of $x F_3$ is predicted with the rise of t when x is small. This is consistent with the experimental findings. A close look at the

formula (18) would reveal that for relatively large x our formula predicts a decrease of $x F_3$ with increasing Q^2 . As can be seen in Fig. 1 this is entirely in agreement with the experimental results. As mentioned earlier, in deriving our results we did not use the results of LC-RG and hence we conclude that the scaling behaviour is built in inside the notion of asymptotic freedom. The fact that earlier derivations LC-RG had given similar results points out an inner consistency of the formalism and particularly indicates the coherence of RG equation and asymptotic freedom conjecture.

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